Maximizing Area
Rectangular Gardens: Example 2

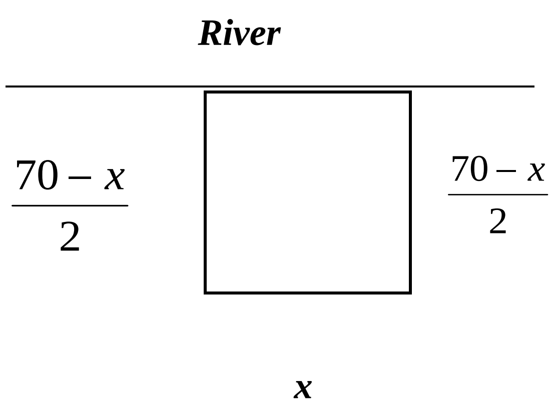
Names: ***Key***____
Date: ____________

About this Laboratory

In this laboratory, we will explore the concept of maximizing the area when a fixed amount of material is given to use for the perimeter. In the first example, you are asked to explore the dimensions needed for the border of a rectangular garden so that the area of the garden is maximized. In the second example, you are asked to explore the dimensions needed if the rectangular garden is bordered on one side by a river. Remember that all graphical solutions are only approximations.

Example 2:
You are given 70 meters of fencing to construct a rectangular garden with one side along a river, so no fencing is needed for that side.

A) Sketch a picture of the garden and let x represent the width of the side parallel to the river.
B) Given 70 meters of fencing, define the length in terms of the width, x.
C) Label only the sides of your rectangle that have fencing, in terms of x.



width: x

length: $\frac{70-x}{2}$

Find the equation for the area in terms of the width x. Remember that the rectangular garden has a border along a river, so 70 meters of fencing is used for *three* sides of the rectangle. Because x is the independent variable, you should only have numbers and the variable x in the right-hand side of the equation.

$$A = x\left(\frac{70 - x}{2}\right) = 35x - \frac{1}{2}x^2$$

Examine your equation and then circle the type of graph that represents the equation.

Line Parabola Cubic Circle

Why did you choose your selection of graph type?

The equation A= $35x - \frac{1}{2}x^2$ ***is quadratic. The graphs of quadratic equations are parabolas.***

Enter 70 into the white rectangle in the applet next to the word graph, and then choose **Set**. Point to a side and then hold down the left mouse button to drag and form a new figure. Release the button and choose **Record**. Repeat this process until you have at least 10 values in the applet table.

Defend that in all cases when you **Record**, the amount of fencing used is 70 meters.

The width is x and the length is $\frac{70 - x}{2}$***. In all cases,***

$$x + \frac{70 - x}{2} + \frac{70 - x}{2} = 70.$$

Once you have at least 10 entries, copy your values into the given table. Notice that two unique entries should have zero as the related area. If you did not find them using the applet, look at your equation to help determine those widths.

Width	Area
0	0
2	***68***
6	***192***
12	***348***
22	***528***
32	***608***
42	***588***
52	***468***
60	***300***
70	0

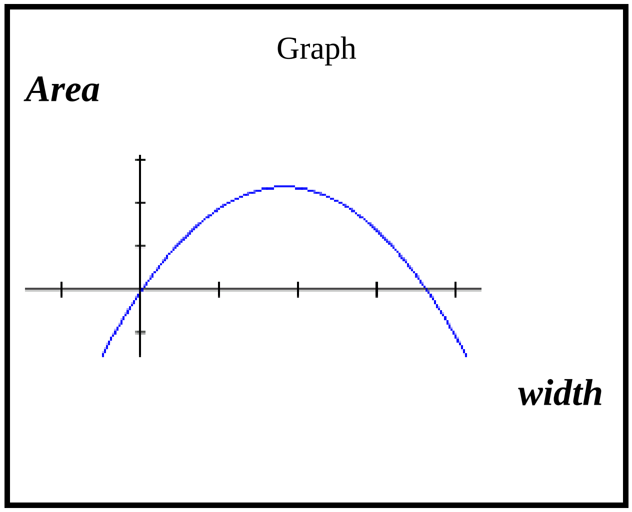
Explain how the values in the table help to confirm the type of graph that you expect the area equation to represent.

As the width values increase from zero, the area values first increase and then decrease.

How can you use the table to find the x-intercepts?

x-intercepts occur when the "y" value is zero. Find the x-values that are associated with area values of zero.

In the applet, select **Graph** and **Round** and then **Equat.**
In the rectangular box next to Y1, enter the right-hand side of your Area equation (in terms of x).



HINT: You MUST use the * symbol for all multiplication. For instance, 2(3x + 5) must be entered as 2*(3*x + 5).

Once your equation is complete press the **ENTER** key and then select **OK.** Sketch your graph in the space provided. Label the axes with the appropriate *terms* (words such as length, width, area, volume, ...).

What does the x variable represent in this example? ***width in meters***

What does the y variable, or A, represent in this example? ***area in square meters***

What is the domain for the equation of the function? ***The set of real numbers***

What is the range for the equation of the function? $\boldsymbol{A \leq 612.5}$
(Zoom in graphically to approximate an answer and then come back later to verify your results.)

What x values make sense for the problem? $\boldsymbol{0 \leq x \leq 70}$
(This is the restricted domain. You might wish to zoom in graphically to approximate an answer and then come back later to verify your results.)

What y values make sense for the problem? $\boldsymbol{0 \leq A \leq 612.5}$
(This is the restricted range. You might wish to zoom in graphically to approximate an answer and then come back later to verify your results.)

Zoom in graphically to approximate each x-intercept and then come back to confirm your results.

x-intercept: (***0*** , 0)
x-intercept: (***70*** , 0)

Confirm the x-intercepts *algebraically* by using the equation for the area of the pen.

$A = x\left(\frac{70-x}{2}\right)$	***0 = x or***	***0 =*** $\frac{70-x}{2}$
$0 = x\left(\frac{70-x}{2}\right)$		***0 = 70 - x***
		x = 70
	(0, 0)	***(70, 0)***

What do the values of the coordinates of the x-intercepts mean in this real life problem?
(To answer the question, THINK: What is so important about these **coordinates**? What do **they** mean in terms of your units?)

The coordinates of the x-intercepts tell us which values of width have an area of zero. They also give the boundaries for the restricted domain.

Now move the cursor and select the vertex of the parabola on your screen.
Zoom in graphically to approximate the coordinates of the vertex and then come back to confirm you answer.
(***35*** , ***612.5***)

Confirm the coordinates of the vertex *algebraically* by using the equation for the area of the pen.

$$A = -\frac{1}{2}x^2 + 35x$$

$$A(35) = -\frac{1}{2}(35)^2 + 35(35)$$

Vertex: $\left(\frac{-b}{2a}, A\left(\frac{-b}{2a} \right) \right)$

= 612.5

$$\frac{-b}{2a} = \frac{-35}{2\left(-\frac{1}{2}\right)} = 35$$

(35, 612.5)

Describe the real life meaning of the vertex of the parabola.
(THINK: What do **the coordinates** mean in terms of your units? Where is the vertex in relationship to the other points on the graph?)

The coordinates of the vertex tell us that when the width is 35 meters, the maximum area of 612.5 square meters is obtained.

Fill in the table. Select **Table** and enter values for "x".

x	A(x)
-10	***-400***
0	***0***
10	***300***
20	***500***
30	***600***
40	***600***
50	***500***
60	***300***
70	***0***
80	***-400***

Explore the values of x and A in the table and explain how *this table* can be used to predict the maximum area and restricted x-values found earlier. **(Be careful how you use the table to explain your answer. Hint: Explain what happens to the x and A values around the x-intercepts and around the vertex of the parabola.)**

Due to the symmetry found in the parabola, the table allows us to determine that the vertex will occur at x = 35 and that the maximum area will be greater than 600. Since the values of x = 0 and x = 70 yield an area of zero, they determine the boundaries for the restricted domain.

What dimensions (**length and width**) will give the maximum area?
35 meters x 17.5 meters
(Be sure to use the appropriate units in your answer.)

What type of figure gives the maximum **area**? ***a rectangle***

What is the maximum **area**? ***612.5 square meters***
(Be sure to use the appropriate units in your answer.)